9.4: 6, 14, 20, 22, 26

6. $|\mathbf{r}| = \sqrt{4^2+4^2} = 4\sqrt{2}$ ft. A line drawn from the point P to the point of application of the force makes an angle of $180^\circ - (45+30)^\circ = 105^\circ$ with the force vector. Therefore, $|\boldsymbol{\tau}| = |\mathbf{r}\times\mathbf{F}| = |\mathbf{r}|\,|\mathbf{F}|\sin\theta = \left(4\sqrt{2}\right)(36)\sin 105^\circ \approx 197$ ft-lb.

14. We know that the cross product of two vectors is orthogonal to both. So we calculate

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 1 & 1 \\ 2 & 0 & 1 \end{vmatrix} = \begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix}\mathbf{i} - \begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix}\mathbf{j} + \begin{vmatrix} 1 & 1 \\ 2 & 0 \end{vmatrix}\mathbf{k} = \mathbf{i}+\mathbf{j}-2\mathbf{k}$$

Thus, two unit vectors orthogonal to both are $\pm\frac{1}{\sqrt{6}}\langle 1,1,-2\rangle$, that is, $\left\langle \frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, -\frac{2}{\sqrt{6}}\right\rangle$ and $\left\langle -\frac{1}{\sqrt{6}}, -\frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}}\right\rangle$.

20. Since $|\mathbf{u}\times\mathbf{v}| = |\mathbf{u}|\,|\mathbf{v}|\sin\theta, 0\le\theta\le\pi$, $\mathbf{u}\times\mathbf{v}|$ achieves its maximum value for $\sin\theta = 1 \Rightarrow \theta = \frac{\pi}{2}$, in which case $|\mathbf{u}\times\mathbf{v}| = |\mathbf{u}|\,|\mathbf{v}| = 15$. The minimum value is zero, which occurs when $\sin\theta = 0 \Rightarrow \theta = 0$ or π, so when $\mathbf{u}$, $\mathbf{v}$ are parallel. Thus, when $\mathbf{u}$ points in the same direction as $\mathbf{v}$, so $\mathbf{u} = 3\,\mathbf{j}$, $|\mathbf{u}\times\mathbf{v}| = 0$. As $\mathbf{u}$ rotates counterclockwise, $\mathbf{u}\times\mathbf{v}$ is directed in the negative z-direction (by the right-hand rule) and the length increases until $\theta = \frac{\pi}{2}$, in which case $\mathbf{u} = -3\,\mathbf{i}$ and $|\mathbf{u}\times\mathbf{v}| = 15$. As $\mathbf{u}$ rotates to the negative y-axis, $\mathbf{u}\times\mathbf{v}$ remains pointed in the negative z-direction and the length of $\mathbf{u}\times\mathbf{v}$ decreases to 0, after which the direction of $\mathbf{u}\times\mathbf{v}$ reverses to point in the positive z-direction and $|\mathbf{u}\times\mathbf{v}|$ increases. When $\mathbf{u} = 3\,\mathbf{i}$ (so $\theta = \frac{\pi}{2}$), $|\mathbf{u}\times\mathbf{v}|$ again reaches its maximum of 15, after which $|\mathbf{u}\times\mathbf{v}|$ decreases to 0 as $\mathbf{u}$ rotates to the positive y-axis.

22. $$\mathbf{a}\cdot(\mathbf{b}\times\mathbf{c}) = \begin{vmatrix} 1 & 1 & -1 \\ 1 & -1 & 1 \\ -1 & 1 & 1 \end{vmatrix} = 1\begin{vmatrix} -1 & 1 \\ 1 & 1 \end{vmatrix} - 1\begin{vmatrix} 1 & 1 \\ -1 & 1 \end{vmatrix} + (-1)\begin{vmatrix} 1 & -1 \\ -1 & 1 \end{vmatrix} = -2-2+0 = -4.$$

So the volume of the parallelepiped determined by $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c}$ is $|-4| = 4$ cubic units.

26. $\mathbf{u} = \overrightarrow{AB} = \langle 2,-4,4\rangle$, $\mathbf{v} = \overrightarrow{AC} = \langle 4,-1,-2\rangle$ and $\mathbf{w} = \overrightarrow{AD} = \langle 2,3,-6\rangle$.

$$\mathbf{u}\cdot(\mathbf{v}\times\mathbf{w}) = \begin{vmatrix} 2 & -4 & 4 \\ 4 & -1 & -2 \\ 2 & 3 & -6 \end{vmatrix} = 2\begin{vmatrix} -1 & -2 \\ 3 & -6 \end{vmatrix} - (-4)\begin{vmatrix} 4 & -2 \\ 2 & -6 \end{vmatrix} + 4\begin{vmatrix} 4 & -1 \\ 2 & 3 \end{vmatrix} = 24-80+56 = 0,$$ so the volume of the parallelepiped determined by $\mathbf{u}$, $\mathbf{v}$ and $\mathbf{w}$ is 0, which says these vectors lie in the same plane. Therefore, their initial and terminal points A, B, C and D also lie in the same plane.